

# Reachability-based confidence-aware probabilistic collision detection in highway driving

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**SAFE-UP** 

MFTS, Dresden, Dec 2022

## Background

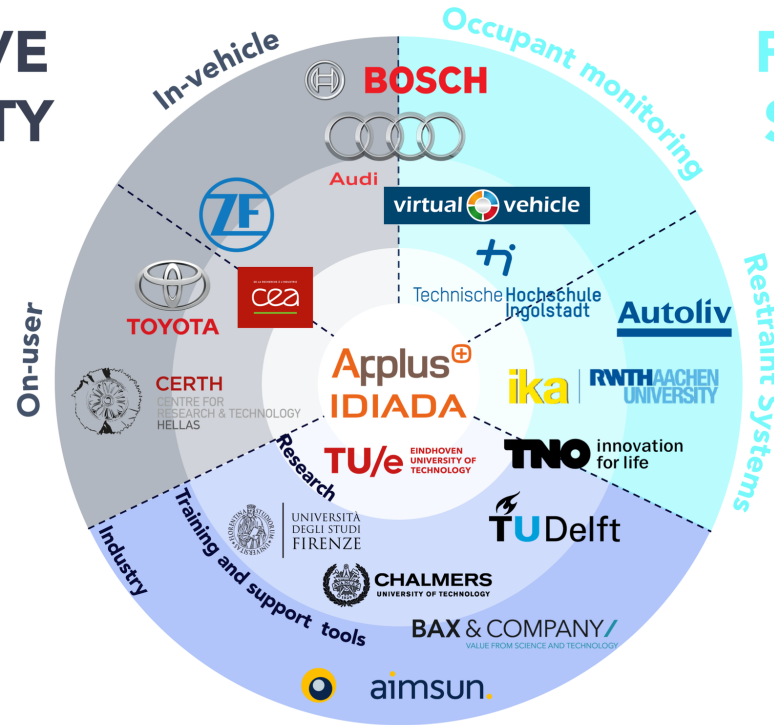
# SAFE-UP

## ACTIVE SAFETY

**SAFE-UP** aims to proactively address the upcoming safety challenges based on 3 key pillars: i) future safety-critical scenarios ii) new safety technologies and iii) novel safety assessment methodologies.

TUD related subtasks:

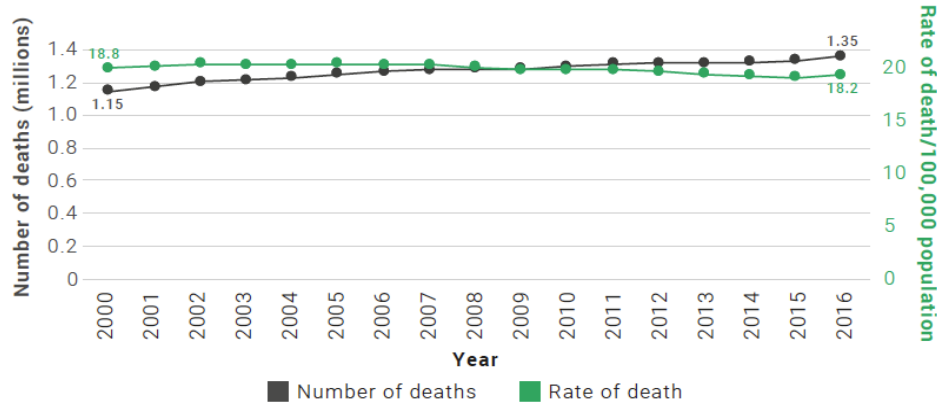
**Safety-critical scenario identification**



## FUTURE SCENARIOS

## Safety is of pivotal importance for (automated) driving.

### Road deaths from WHO



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**Self-driving cars**  
**Google's self-driving car in broadside collision after other car jumps red light**

Autonomous Lexus SUV could not prevent accident that caved in front and rear passenger-side doors, setting off airbags and forcing it to be towed away

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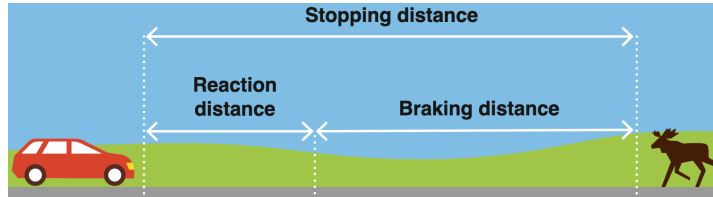
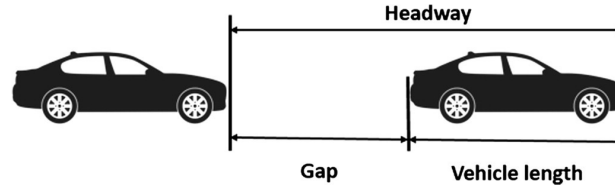
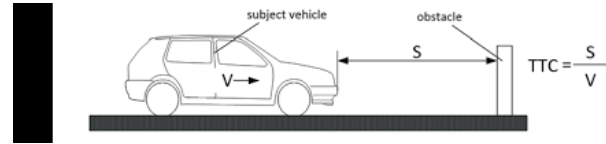
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## Motivation

- Time based
  - Time to Collision (TTC)
  - Time to Headway
- Distance based
  - Stopping distance



*One-dimensional and deterministic metrics cannot address motion uncertainties.*

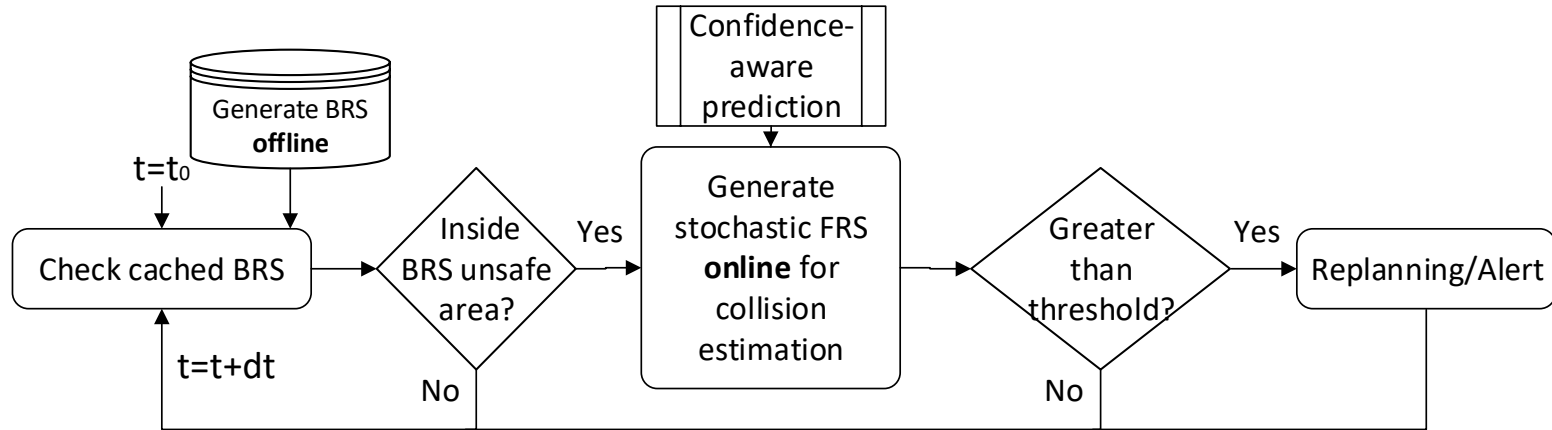
*How to address motion uncertainties?  
How to ensure safety?*

## Goal

*How to ensure safety?*

*How to address uncertainties?*

To develop a two-level **reachability-based confidence-aware** collision detection framework



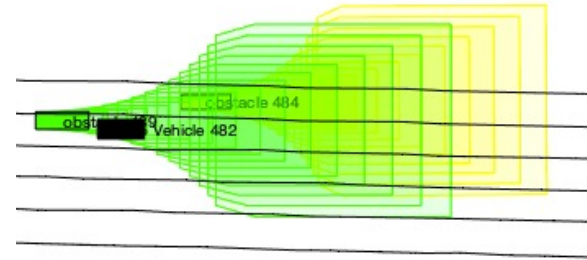
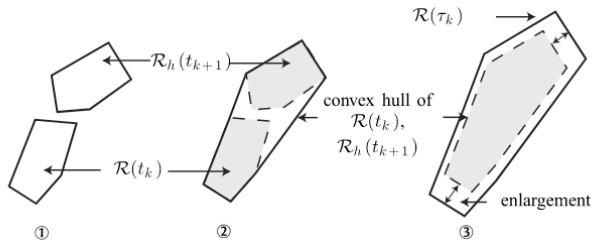
BRS: Backward reachable set

FRS: Forward reachable set

Stochastic FRS, where each state has assigned a possibility

Assume ego has a planned trajectory

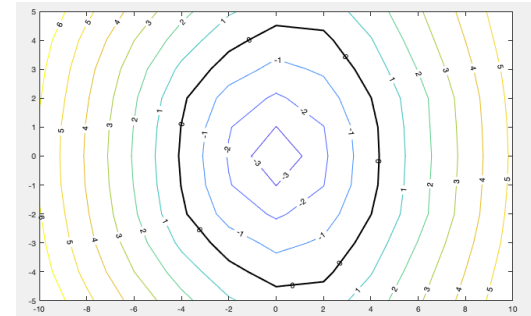
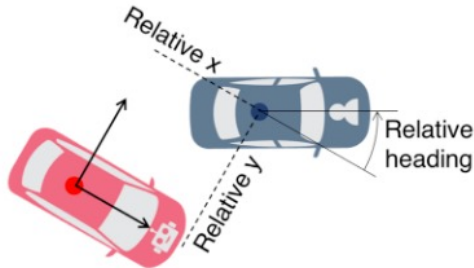
- **Forward Reachable Set (FRS)**  
(M. Althoff from TU Munich, Germany)
  - Consider max acc capacity
  - Formal safety verification



### ○ Backward Reachable Set (BRS)

(C. Tomlim from Berkeley, USA)

- Pursuit-evasion game -> Hamilton Jacobi partial differential equation (HJ PDE) -> Solve PDE to check if crash occurs
- Offline computation by discretising states: speed, position, and angle
- Online use: a look-up table
- Formal safety verification

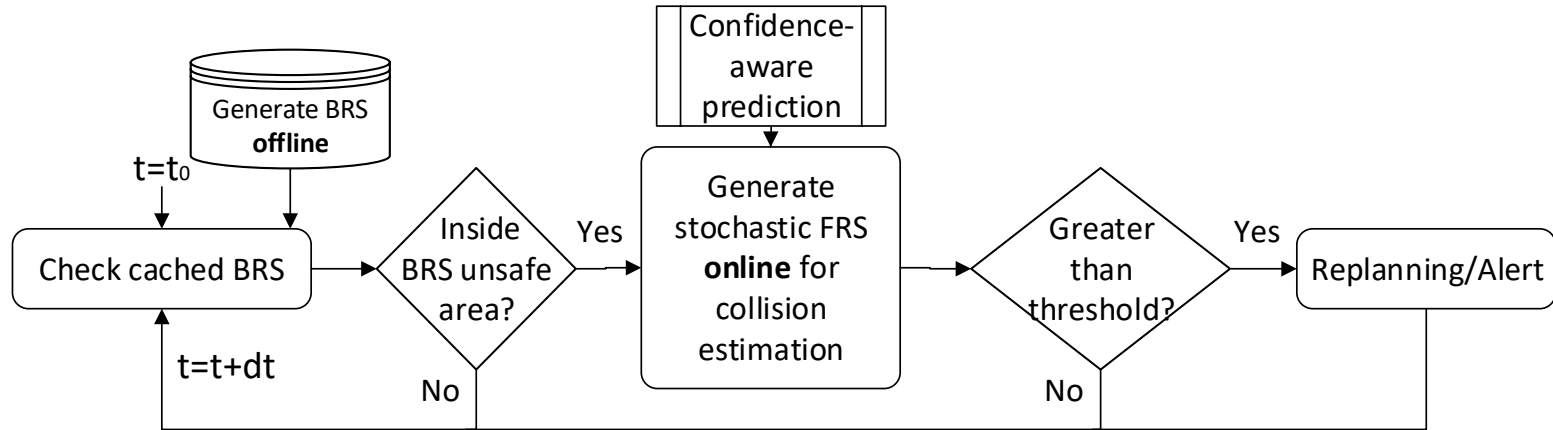


Sliced zero-level BRS subset, within which the sur vehicle can hit the ego (origin)

*BRS is still over conservative due to the worst interaction assumption.*

## Goal

# To develop a two-level **reachability-based confidence-aware** collision detection framework



BRS: Backward reachable set

FRS: Forward reachable set

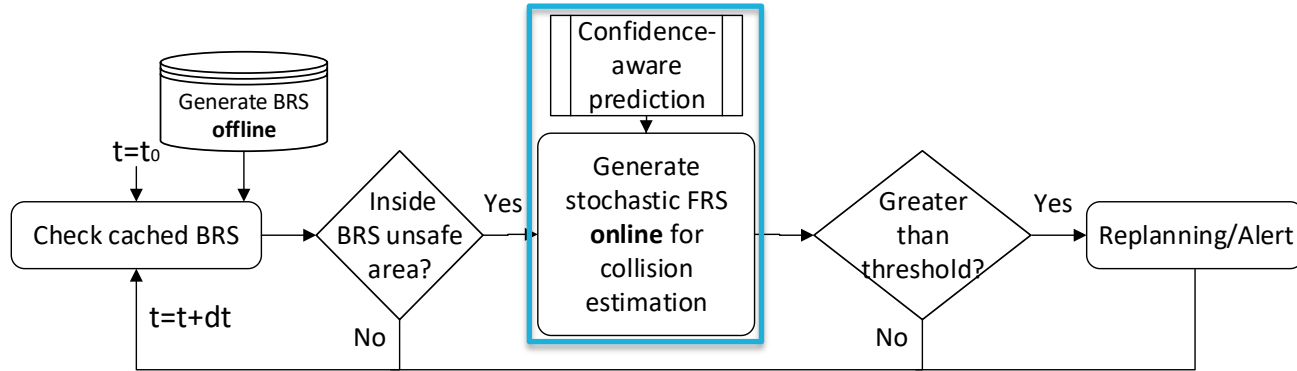
Stochastic FRS, where each state has assigned a possibility

Assume ego has a planned trajectory



## Goal

To develop a two-level **reachability-based confidence-aware** collision detection framework



- **Key components** for stochastic FRS
  - State transition matrix (Base)
  - Learning-based multi-modal prediction (Input)
  - Confidence-aware concept (Infuse to the input prediction)

State transition matrix  $\Phi$  Althoff et al. (2010)

- Discretised time and state  $p_i(k)$
- Deterministic heuristic rules for discretised input probability (Markovian)

$$\Phi_{ji} = \sum_{u \in \mathcal{U}} \Phi_{ji}^u$$

- State transition  $\mathbf{p}(k+1) = \Phi \cdot \mathbf{p}(k)$   
offline

*Real vehicle motions are not Markovian! The real control input not only depends on current state, but also previous states and sur environment*

### Multi-modal prediction to determine input probability for $\Phi$

- Non-Markovian process, using previous tracks for prediction
  - A multi-modal **input** predictor (variation of previous work, Wang et al., 2022)
- Output probabilistic input distribution at each predicted time step, then do integrals to calculate  $\Phi_{ji}^u(k)$

$$\Phi_{ji}(k) = \sum_{u \in \mathcal{U}} \Phi_{ji}^u(k)$$

$$\mathbf{p}(k+1) = \underset{\text{online}}{\Phi}(k) \cdot \mathbf{p}(k)$$

*Results largely depend on the prediction accuracy!*

## Confidence-aware prediction

- A Bayesian belief vector for different distributions

Single normal distribution

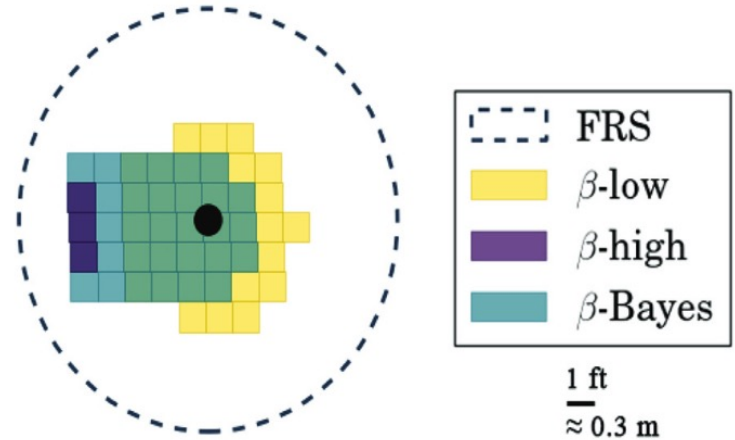


Multiple confidence-aware normal distributions

$$\beta = [1/2, 1, 2]$$

$$b^0(\beta) = [1/3, 1/3, 1/3]$$

- To address the **predictor performance**, the belief vector is updated with posterior estimation



Confidence-aware Q-value pedestrian prediction (Fridovich-Keil et al., 2020)

## Results: Stochastic FRS

HSRS: Stochastic FRS via heuristic rules (baseline)

PSRS: Prediction based stochastic FRS

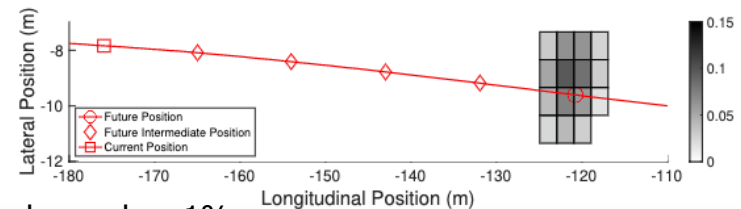
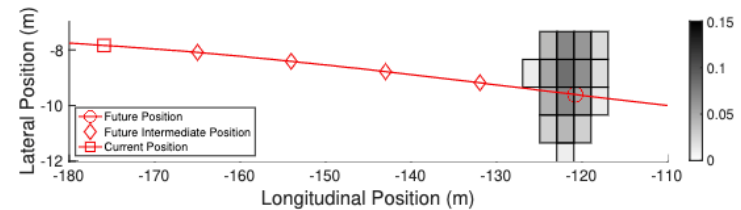
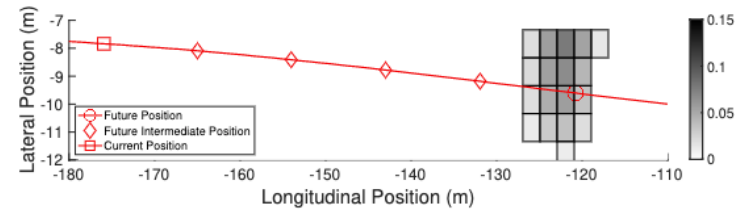
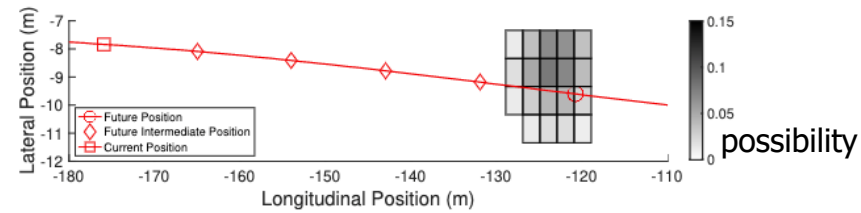
PSRS- $3\beta$ : Prediction based stochastic FRS using  $3\beta$

$\beta = [1/2, 1, 2]$

PSRS- $5\beta$ : Prediction based stochastic FRS using  $5\beta$

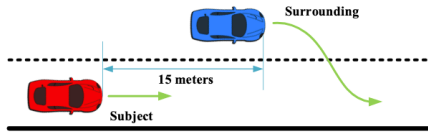
$\beta = [1/3, 1/2, 1, 2, 3]$

(Predict 5 time steps in 2 seconds)



\*only display prob > 1%

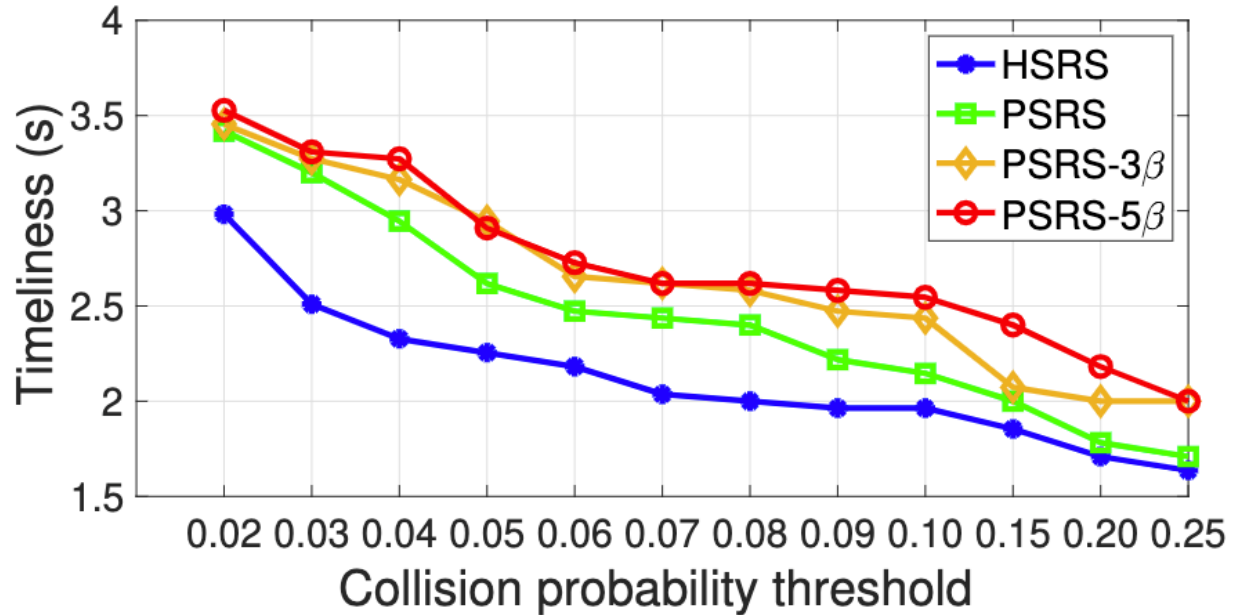
## Results: Stochastic FRS



Simulated risky cut-in events

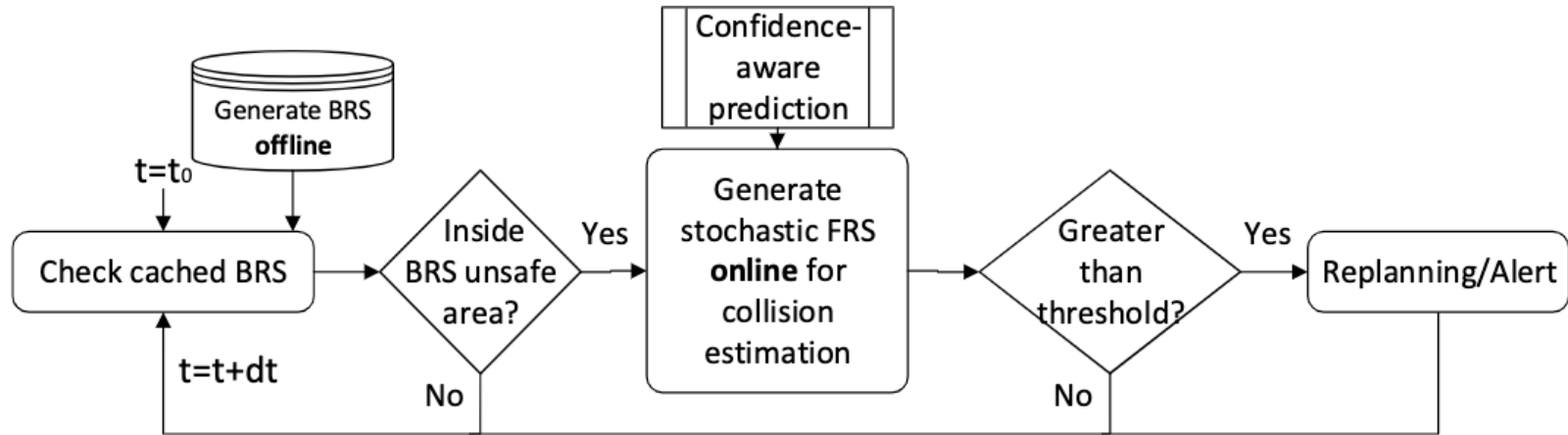
$V \sim [20, 35]$

33 out of 256 crash cases



Average performance for 33 crash events with different thresholds. Timeliness is the time the crash occurs after the collision probability reaches the threshold.

## Results: Integrated framework



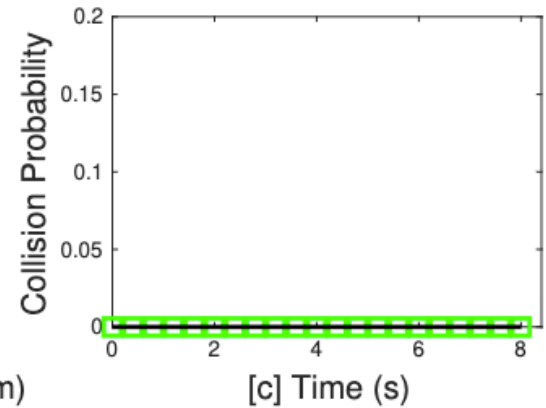
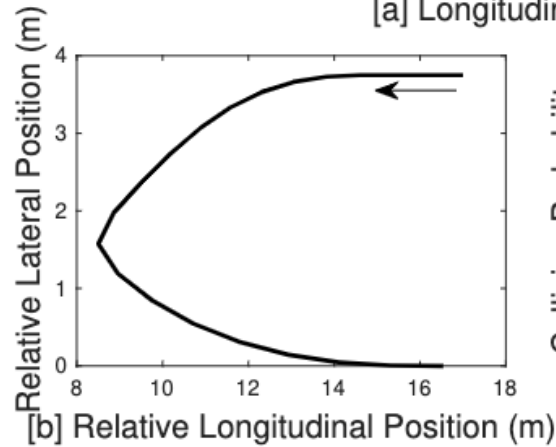
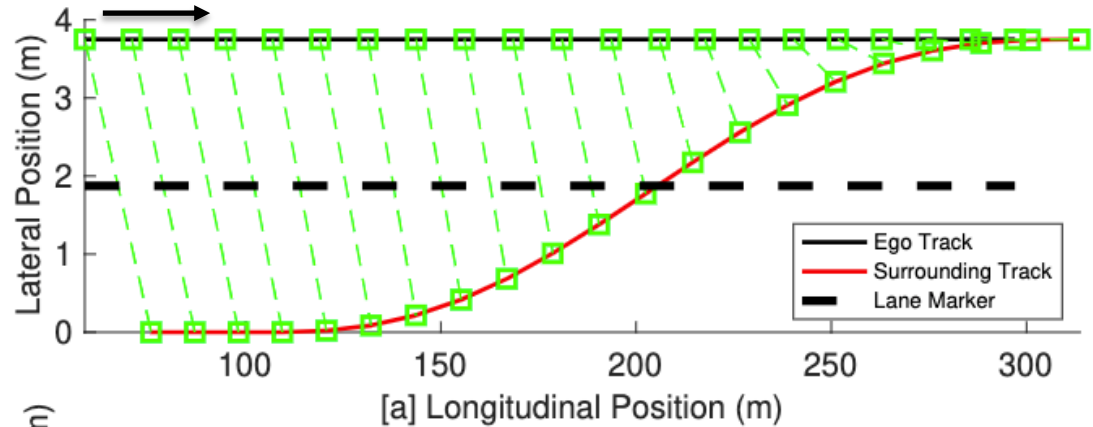
Validate the proposed collision detection framework in both risky and non-risky events

## Results: Integrated framework

Simulated cut-in non-risky event

$$V_{\text{sub}} = 30 \text{ m/s}, V_{\text{sur}} = 28 \text{ m/s}$$

Green: theoretically safe by the BRS.



\*dash lines connect vehicle positions at the same time step



## Results: Integrated framework

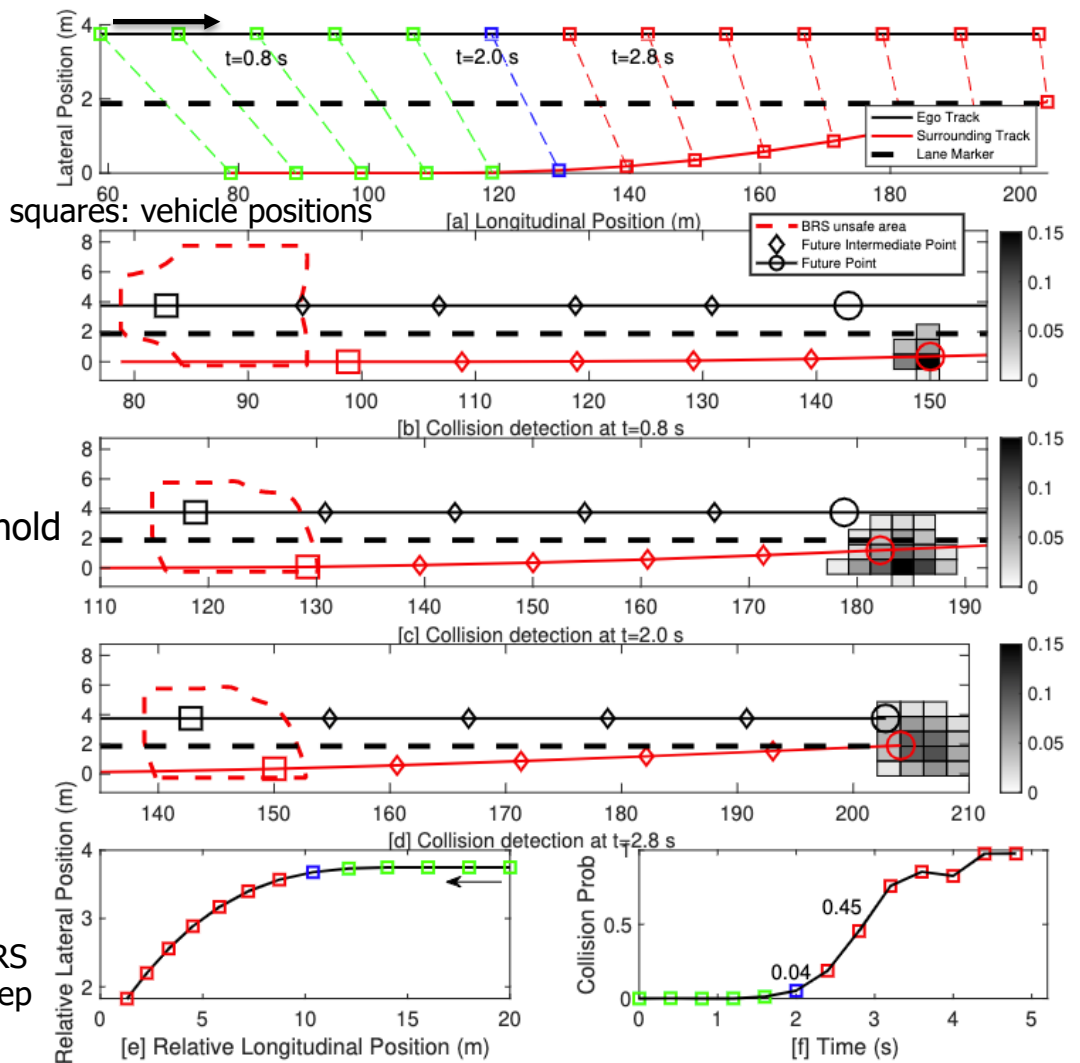
Simulated cut-in risky event  
(lead to crash at 5 seconds)

$$V_{sub} = 30 \text{ m/s}, V_{sur} = 25 \text{ m/s}$$

Green: theoretically safe by the BRS.

Blue: collision probability below a threshold  
(0.05 in this case).

Red: above the threshold. **Unsafe.**



\*only show the stochastic FRS  
at the last prediction time step  
for convenience

1. Developed a two-level reachability-based framework for collision detection
  - Safety can be ensured in non-risky events
2. To improve collision probability estimation, established a prediction-based confidence-aware stochastic FRS.

## Future work

- 1. Implement the proposed safety assessment approaches for real-world test. Further simplify the computation of stochastic FRS.*
- 2. The risk assessment is for posterior analysis. Further investigate the integration of motion planning and risk assessment*

Khaled Alaa, et al. "Probabilistic Risk Assessment for Chance-Constrained Collision Avoidance in Uncertain Dynamic Environments." submitted to *ICRA 2023*.

# Thanks for listening!

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- Fridovich-Keil, et al. "Confidence-aware motion prediction for real-time collision avoidance." *The International Journal of Robotics Research* 39.2-3 (2020): 250-265.
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- Wang, et al. "Prediction-Based Reachability Analysis for Collision Risk Assessment on Highways." *IEEE Intelligent Vehicles Symposium* (2022).

## Use risk assessment for safer and more efficient motion planning.

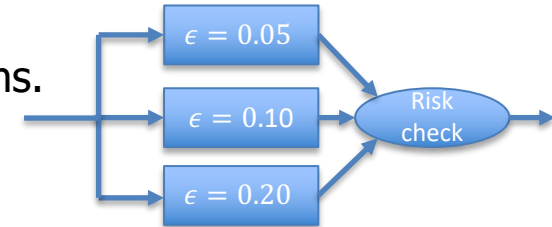
- A scenario-based chance-constrained motion planner (de Groot et al., 2021)
  - with a parameter: risk upper bound
  - the bound is not tight

Upper Bound	Max CP
$\epsilon = 0.05$	0.0094
$\epsilon = 0.1$	0.0456
$\epsilon = 0.2$	0.0727

A robot shares dynamic environments with 6 pedestrians.

Hybrid $\epsilon = \{0.05, 0.1, 0.2\}$	0.0454
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- 1) Plan three trajectories in parallel
- 2) Do one-shot risk check (Simply integrate prob density func for collision estimation)
- 3) Pick the least conservative plan as long as its risk below the CP threshold.



<0.05

# Probabilistic Risk Assessment for Chance-Constrained Collision Avoidance in Uncertain Dynamic Environments

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